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RHEOLOGICAL BEHAVIOR OF A DILUTE SUSPENSION OF RELATIVELY COARSE DEFORMABLE PARTICLES IN A SIMPLE SHEAR FLOW

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The behavior of dilute suspensions of stiff and deformable ellipsoidal particles of such a size that it is necessary to take account of the influence of Brownian forces on the particle behavior is investigated in [1, 2]. The case is considered in this paper when the suspended deformable particles are relatively coarse, i.e., the influence of the Brownian and inertial forces on the microstructure behavior can be neglected. The rheological behavior of the suspension is here determined by the microstructure behavior under the action of just hydrodynamic forces.

It is shown in [3] that during simple shear the stiff ellipsoidal particle subjected to hydrodynamic forces performs periodic motion relative to its center of inertia along one of the closed orbits that form an infinite one-parameter family located on the surface of a sphere. The distribution of the suspended stiff particles over the orbits cannot possibly be determined uniquely without relying on some additional assumptions. Thus, it was considered in [3] that the particles are oriented in such a manner that the principle of minimum energy dissipation is satisfied. The hypothesis about equally probable suspended stiff particle distributions over orbits is examined in [4]. It is assumed in [5, 6] that the axis of rotation of a deformable ellipsoidal particle is in the shear plane. However, as is shown in [7], there are significant discrepancies between experimental data on the macroproperties of the suspension and the theoretical results obtained on the basis of given hypotheses. A method is presented in [8, 9] for finding the distribution of stiff particles over the orbits that is based on assumption of the presence of weak Brownian motion of the particles that does not influence the rheological properties of the suspension. Weak Brownian motion over the lapse of a long time interval results in a certain stationary distribution of the suspended particles over the orbits.

A hypothesis proposed in [8, 9] for stiff particles is used in this paper to find the distribution of deformable particles over the family of orbits. In contrast to the case of relatively coarse stiff particles, the one-parameter family of closed orbits of viscoelastic deformable particles is located on the surface of a triaxial ellipsoid whose geometry and orientation depend on the shear rate, the viscosity of the dispersion medium, and the properties of the suspended particle material.

1. RHEOLOGICAL EQUATIONS OF STATE

Let us consider a dilute suspension of suspended deformable particles. We simulate an element of the suspension microstructure by a deformable ellipsoid of revolution, whose stress state in the material is determined by the internal elasticity G and internal viscosity η. The rheological equations of state for such a suspension have the form [2]

$$T_{ij} = -\langle P\rangle\delta_{ij} + 2\langle\mu_0\rangle d_{ij} + \langle\mu_1 n_i n_j\rangle + \langle\mu_2 n_k n_m n_i n_j\rangle d_{km} + 2\langle\mu_3(d_{ik}n_k n_j + d_{jk}n_k n_i)\rangle. \quad (1.1)$$

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Here T_{ij}, d_{ij} are the stress and strain rate tensors of the suspension, n_i is a vector whose direction agrees with the direction of the particle axis of revolution, and its modulus n equals half the length of this axis, P, μ_i are rheological functions dependent on G, η and n_i, the angular brackets denote particles averaged over angular positions in space and over the lengths of the particle semi-axis of revolution by using distribution function F satisfying the equation

$$\partial F/\partial t + \partial(F\dot{n}_i)/\partial n_i = 0 \tag{1.2}$$

(the dot denotes the derivative with respect to time).

The behavior of an isolated suspended particle is governed by the equations [2]

$$\dot{n}_i = \omega_{ij}n_j + \lambda_1 n_i + \lambda_2 d_{km}n_k n_m n_i + \lambda_3 d_{ij}n_j + \lambda_4\varepsilon_{ijk}M_j n_k + \lambda_5 R_j n_j n_i, \tag{1.3}$$

where λ_i are rheological functions dependent on G, η and the particle geometry, R_j, and M_j are the force and moment of the forces acting on a particle, with the exception of the hydrodynamic forces, ε_{ijk}, δ_{ij} are skew-symmetric and symmetric Kronecker symbols, and ω_{ij} is the velocity vortex tensor.

In the case when it is necessary to take account of the action of Brownian forces on the microstructure [10]

$$R_i = -kT\frac{1}{F}\frac{\partial F}{\partial n_i},\quad M_i = -kT\varepsilon_{ikm}\frac{n_k}{F}\frac{\partial F}{\partial n_m} \tag{1.4}$$

(k is the Boltzmann constant and T is the temperature).

The rheological functions μ_i, λ_i, P for a dilute suspension of suspended particles are found in [2]. However, an error associated with taking incompletely into account all the forces acting on the particle is admitted there in their determination. As is shown in [6] such an error was also admitted in [5] in studying the behavior of suspensions of viscoelastic spheres.

Taking account of the remark mentioned, P, λ_i, and μ_i are determined as follows:

$$\lambda_1 = -\frac{2Gab^2\beta_0''}{\mu\left(2+3ab^2\beta_0''(\eta/\mu-1)\right)}\left(\frac{p}{q_0}\right)^{2/3}\left(1-\frac{q_0}{p}\right),$$

$$\lambda_2 = \frac{1}{a^2}\left(\frac{2}{2+3ab^2\beta_0''(\eta/\mu-1)} - \frac{p^2-1}{p^2+1}\right),$$

$$\lambda_3 = \frac{p^2-1}{p^2+1},\quad \lambda_4 = \frac{3\left(p^2\alpha_0+\beta_0\right)}{16\pi\mu\left(p^2+1\right)},\quad \lambda_5 = \frac{3\beta_0''}{4\pi\mu\left(2+3ab^2\beta_0''(\eta/\mu-1)\right)},$$

$$P = P_0 - \frac{2\mu V}{3}\left\{\frac{\lambda_1}{ab^2\beta_0''} + \frac{n_k n_m}{a^3b^4\beta_0''\alpha_0''}\left(\beta_0''-\alpha_0''+b^2\alpha_0'(\lambda_2 a^2+\lambda_3)\right)d_{km} + kT\left(\frac{1}{a^2}\frac{d}{dn}\left(\frac{p^2\lambda_5}{\beta_0''}\right) - \frac{6\left(p^2-1\right)\lambda_4}{ab^2\left(p^2\alpha_0+\beta_0\right)}\right)\right\},\quad \mu_0 = \mu\left(1+\frac{V}{ab^4\alpha_0'}\right), \tag{1.5}$$

$$\mu_1 = \frac{4GV\left(p/q_0\right)^{2/3}\left(1-q_0/p\right)}{a^2\left(2+3ab^2\beta_0''(\eta/\mu-1)\right)} + \frac{9kTV}{2\pi a^3b^2}\left(\frac{3p(\eta/\mu-1)\frac{d}{dp}\left(ab^2\beta_0''\right)}{2\left(2+3ab^2\beta_0''(\eta/\mu-1)\right)^2} - \frac{1}{2+3ab^2\beta_0''(\eta/\mu-1)} + \frac{1}{2}\frac{p^2-1}{p^2+1}\right),$$

$$\mu_2 = \frac{2\mu V}{a^5b^2}\left(\frac{\alpha_0''+\beta_0''}{b^2\alpha_0'\beta_0''} - \frac{4}{b^2\left(p^2+1\right)\beta_0'} - \frac{2}{\beta_0''\left(2+3ab^2\beta_0''(\eta/\mu-1)\right)}\right),$$

$$\mu_3 = \frac{\mu V}{a^3b^4}\left(\frac{2}{\left(p^2+1\right)\beta_0'} - \frac{1}{\alpha_0'}\right).$$

Here a and b are the semiaxis of revolution and the equatorial radius of the particle, n = a, p = a/b, q_0 is the ratio of the semiaxes of the ellipsoidal particle in the undeformed state, α_0, β_0, α_0', β_0', α_0'', β_0'' are functions of a and b defined in [3], V is the particle volume concentration, μ is the fluid viscosity, and P_0 is the pressure in the fluid in the absence of suspended particles. In the case of a suspension of relatively coarse viscoelastic particles, it is necessary to set T = 0 in (1.5).

2. BEHAVIOR OF AN ISOLATED PARTICLE

Let us investigate the behavior of suspended deformable particles in a simple shear flow

$$V_x = 0,\ V_y = Kx,\ V_z = 0. \tag{2.1}$$

The behavior of a relatively coarse particle in a simple shear flow is described by (1.3). In a coordinate system whose origin is at the particle center of inertia and the coordinate axes are parallel to the axes of the laboratory (x, y, z) coordinate system, (1.3) have the dimensionless form

$$\begin{gathered}(p^2+1)\frac{d\theta}{dt}=\frac{p^2-1}{4}\sin 2\varphi\sin 2\theta,\ (p^2+1)\frac{d\varphi}{dt}=p^2\cos^2\varphi+\sin^2\varphi,\\ \varepsilon\frac{dp}{dt}=-\frac{3ab^2\beta_0''p\,(p/q_0)^{2/3}(1-q_0/p)}{2+3ab^2\beta_0''(\eta/\mu-1)}+\varepsilon\frac{3p\sin 2\varphi\sin^2\theta}{2\left(2+3ab^2\beta_0''(\eta/\mu-1)\right)},\end{gathered} \tag{2.2}$$

wher $\varepsilon = K\mu/G$ is a parameter characterizing the ratio of the hydrodynamic forces to the internal elasticity forces of the particle, θ and φ are the angles of a spherical coordinate system $n_1 = n\cos\varphi\sin\theta$, $n_2 = n\sin\varphi\sin\theta$, $n_3 = n\cos\theta$; and the time scale equals 1/K.

Let us consider the case $\varepsilon < 1$. We assume that at a certain time t = 0 the ellipsoidal particle passes through its undeformed state during motion, i.e.,

$$p = q_0,\ \varphi = \varphi^0,\ \theta = \theta^0 \text{ at } t = 0, \tag{2.3}$$

where φ^0 and θ^0 are unknown angles characterizing the angular location of the particle at the time t = 0 for $p = q_0$.

The system (2.2) describing the orientation and deformation of a suspended particle has, for $\varepsilon = 0$, the periodic solution p_0, φ_0, θ_0:

$$p_0 = q_0,\ \operatorname{tg}\varphi_0 = q_0\operatorname{tg}(tq_0/(q_0^2+1)),\ \operatorname{tg}\theta_0 = Cq_0/(q_0^2\cos^2\varphi_0+\sin^2\varphi_0)^{1/2} \tag{2.4}$$

($C = \tan\theta^0$ is a constant). The solution (2.4) yields the motion of a stiff ellipsoidal particle and agrees with the solution found in [3]. The end of the semiaxis of revolution of a stiff particle performs periodic motion relative to its center of motion in one of the orbits. The orbits are disposed on a sphere surface and form a one-parameter family characterized by the parameter C designated the orbit constant in [3]. It is shown in [11] that the general solution of systems of equations of the type (2.2) is representable in the form of asymptotic series in powers of ε.

Let us assume

$$p=\sum_{m=0}^{\infty}\varepsilon^m p_m,\ \theta=\sum_{m=0}^{\infty}\varepsilon^m\theta_m,\ \varphi=\varphi^0+\sum_{m=0}^{\infty}\varepsilon^m\varphi_m,\ \varphi^0=\sum_{m=1}^{\infty}\varepsilon^m\varphi_m^0. \tag{2.5}$$

The initial conditions for p_m, θ_m and φ_m are

$$p_i = 0,\ \theta_i = 0,\ \varphi_i = \varphi_i^0 \text{ at } i \geqslant 1,\ p_0 = q_0,\ \theta_0 = \theta^0,\ \varphi_0 = 0. \tag{2.6}$$

Executing the usual manipulations, systems of equations can be obtained to determine p_i, φ_i and θ_i. The equations to determine φ_i and θ_i will be differential while the equations for p_i are algebraic because of the singularities of the initial equations (2.2). We find the values of the unknown angles φ_i^0 in such a way as to satisfy the initial conditions (2.6) for p_i.

The zeroth terms of the expansions (2.5) are represented by the relationships (2.4). The first terms of the expansions have the form

$$p_1=\frac{C^2q_0^2}{2\left(ab^2\beta_0''\right)_0}\frac{\sin 2\varphi_0}{\left(C^2q_0^2+q_0^2\cos^2\varphi_0+\sin^2\varphi_0\right)},$$

$$\varphi_1=\frac{1}{\left(q_0^2-1\right)\left(ab^2\beta_0''\right)_0}\left[\frac{\left(2C^2q_0^2+q_0^2+1\right)\left(q_0^2\cos^2\varphi_0+\sin^2\varphi_0\right)}{C^2\left(q_0^4-1\right)}\times\ln\left|\frac{C^2q_0^2+q_0^2\cos^2\varphi_0+\sin^2\varphi_0}{\left(C^2+1\right)\left(q_0^2\cos^2\varphi_0+\sin^2\varphi_0\right)}\right|-\sin^2\varphi_0\right]-\frac{q_0^2-1}{q_0^2}\varphi_1^0\sin^2\varphi_0,$$

$$\theta_1=A\frac{\left(q_0^2\cos^2\varphi_0+\sin^2\varphi_0\right)^{1/2}}{C^2q_0^2+q_0^2\cos^2\varphi_0+\sin^2\varphi_0},\ A=\frac{q_0^2\left(C^2\left(q_0^2+1\right)+2\right)}{C\left(ab^2\beta_0\right)_0\left(q_0^2-1\right)^2}\operatorname{arctg}\left(\frac{\operatorname{tg}\varphi_0}{q_0}\right)-$$

$$-\frac{2q_0\left(C^2q_0^2+1\right)^{1/2}\left(C^2+1\right)^{1/2}}{C\left(ab^2\beta_0''\right)_0\left(q_0^2-1\right)^2}\operatorname{arctg}\left(\frac{\left(C^2q_0^2+1\right)^{1/2}\operatorname{tg}\varphi_0}{\left(C^2+1\right)^{1/2}q_0}\right)+$$

$$+\frac{C\left(q_0^2+\left(q_0^2-1\right)^2\varphi_1^0\left(ab^2\beta_0''\right)_0\right)}{2\left(ab^2\beta_0''\right)_0\left(q_0^2-1\right)q_0}\sin 2\varphi_0+\frac{q_0\left(2C^2q_0^2+q_0^2+1\right)}{2C\left(ab^2\beta_0''\right)_0\left(q_0^4-1\right)}\times$$

$$\sin 2\varphi_0\ln\left|\frac{C^2q_0^2+q_0^2\cos^2\varphi_0+\sin^2\varphi_0}{\left(C^2+1\right)\left(q_0^2\cos^2\varphi_0+\sin^2\varphi_0\right)}\right|,\quad \varphi_1^0=\frac{q_0^2}{q_0^2+1}\,\frac{2+3\left(ab^2\beta_0''\right)_0(\eta/\mu-1)}{3\left(ab^2\beta_0''\right)_0} \tag{2.7}$$

[$(ab^2\beta_0'')_0$ is the value of the function $ab^2\beta_0''$ for $p = q_0$]. If the suspended viscoelastic particle is spherical in shape in the undeformed state, i.e., $q_0 = 1$, then

$$p_1=\frac{15C^2}{8\left(C^2+1\right)}\sin 2\varphi_0,\quad \varphi_1=\frac{15C^2}{32\left(C^2+1\right)}\sin 2\varphi_0,$$

$$\theta_1=\frac{15C^3}{16\left(C^2+1\right)^2}\left(\varphi_0-\frac{\sin 4\varphi_0}{4}\right),\quad \varphi_1^0=\frac{5}{4}+\frac{1}{2}\left(\frac{\eta}{\mu}-1\right).$$

Approximations of higher order in ε can be obtained analogously.

As follows from (2.4) and (2.7), the deformations and orientations of the suspended particle occur in such a manner that the end of the semiaxis of rotation of the particle performs periodic motion relative to its center of inertia along one of the infinite one-parameter family of closed orbits characterized by C. The one-parameter family of orbits is on the surface of a triaxial ellipsoid whose geometry and orientation depend on the shear rate, the internal properties of the particle material, and the fluid viscosity. The angle φ^0 characterizes the orientation of this triaxial ellipsoid of orbits relative to a coordinate system whose origin is related to the center of inertia of the suspended particle.

When the suspended particle is a sphere in the undeformed state, these results agree with the solutions [6].

3. RHEOLOGICAL BEHAVIOR OF THE SUSPENSION

To determine the rheological behavior of the suspension, the distribution function over the angular positions and lengths of the particle semiaxes of rotation must be found that satisfies (1.2). Since the end of the particle semiaxis of rotation is on the surface of the orbit triaxial ellipsoid during motion in a simple shear flow, then it is convenient to go over to Gauss coordinates (C, τ) on the surface of the orbit ellipsoid and order to determine the function F. The passage from the spherical (n, θ, φ) coordinate system to the Gauss coordinates (C, τ) is realized by means of the formulas

$$n = Z(C, \tau),\ \varphi = \varphi(C, \tau),\ \theta = \theta(C, \tau),$$

where $Z = ab^2p^{2/3}$; $\tau = q_0t/(q_0^2 + 1)$; p, φ and θ are solutions of the equations of particle motion obtained from (2.4), (2.5), and (2.7).

Taking (1.3) and (1.4) into account, (1.2) for the determination of the stationary distribution of the suspended particles in orientation and deformation spaces in a simple shear flow (2.1) has the form

$$\operatorname{div}(F\dot{\mathbf{n}})+\operatorname{div}(kT(\lambda_4-\lambda_5)(\mathbf{n}\cdot\operatorname{grad}F)\mathbf{n})+\operatorname{div}(kT\lambda_4n^2\operatorname{grad}F)=0. \tag{3.1}$$

Here the last two terms characterize the influence of Brownian motion on the behavior of the suspension microstructure, $\mathbf{n}$ can change only under the action of hydrodynamic forces $\dot{n}_i = \omega_{ij}n_j+\lambda_1n_i+\lambda_2d_{km}n_kn_mn_i + \lambda_3d_{ij}n_j$. The function F should satisfy the condition of normalization to one.

In the case of relatively coarse particles, when the influence of Brownian motion of the particles on the rheological behavior of the suspension, meaning also the last two terms in (3.1), can be neglected, (3.1) has the following form in the Gauss (C, τ) coordinate system

$$(\partial/\partial\tau)(F\sqrt{a}) = 0, \tag{3.2}$$

where $a = \det(a_{ij})$, and a_{ij} is the metric tensor of a curvilinear nonorthogonal (C, τ) coordinate system on the surface of the triaxial orbit ellipsoid:

$$a_{11} = Z_C{}^2 + Z^2(\theta_C{}^2 + \sin^2\theta\varphi_C{}^2),\ a_{22} = Z_\tau{}^2 + Z^2(\theta_\tau{}^2 + \sin^2\theta\varphi_\tau{}^2),$$

$$a_{12} = a_{21} = Z_CZ_\tau + Z^2(\theta_C\theta_\tau + \sin^2\theta\varphi_C\varphi_\tau);$$

where the subscripts C and τ denote differentiation of the appropriate functions with respect to these variables.

The solution (3.2) is found to the accuracy of an unknown function f(C) that characterizes the distribution of the suspended particles over the orbits

$$F(C, \tau) = f(C)/\sqrt{a}. \tag{3.3}$$

To determine f(C) we will assume that weak Brownian motion exists that does not influence the rheological properties of the suspension. But the action of forces due to weak Brownian motion of the particles results in the limit of a long time interval in a certain steady particle distribution over the orbits. Let us integrate (3.1) over a certain simply-connected domain σ belonging to the surface of the orbit ellipsoid by applying the Green's formula [12] to go from a surface integral over to an integral over the closed curve L bounding σ and taking into account that ($\mathbf{n}\cdot\operatorname{grad} F$) , we obtain

$$\int_L (\dot{\mathbf{n}}\cdot\boldsymbol{\alpha}) F\, dl + \int_L kT\lambda_4 n^2 (\boldsymbol{\alpha}\cdot\operatorname{grad} F)\, dl = 0 \tag{3.4}$$

($\boldsymbol{\alpha}$ is the exterior normal to L located in the tangent plane to the orbit ellipsoid).

Let us select the simply-connected domain σ so that the curve L would be an orbit characterized by the constant C. Then taking $(\boldsymbol{\alpha}\cdot\dot{\mathbf{n}}) = 0$ into account, (3.4) has the form

$$\int_C \lambda_4 n^2 (\boldsymbol{\alpha}\cdot\operatorname{grad} F)\, dl = 0$$

or

$$\int_0^{2\pi} \frac{\lambda_4 Z^2 a_{22}}{a}\, d\tau \frac{df}{dC} + \int_0^{2\pi} \frac{\lambda_4 Z^2}{\sqrt{a}} \left[a_{22}\frac{\partial}{\partial C}(a^{-1/2}) - a_{12}\frac{\partial}{\partial \tau}(a^{-1/2})\right] d\tau f(C) = 0,$$

whose solution we write as

$$f(C) = \mathrm{const}\cdot\exp\left\{-\int_0^C \left[\int_0^{2\pi} \frac{\lambda_4 Z^2}{\sqrt{a}} \left(a_{22}\frac{\partial}{\partial C}(a^{-1/2}) - a_{12}\frac{\partial}{\partial \tau}(a^{-1/2})\right) d\tau \bigg/ \int_0^{2\pi} \frac{\lambda_4 Z^2 a_{22}}{a}\, d\tau\right] dC\right\}. \tag{3.5}$$

The constant in (3.5) is determined from the condition of normalization of the function F(C, τ):

$$\int_0^\infty f(C)\, dC = \frac{1}{4\pi}.$$

Averaging any value $\gamma(C, \tau)$ entering in the rheological equations of state (1.1) is performed as follows

$$\langle\gamma(C,\tau)\rangle = 2\int_0^\infty f(C) \int_0^{2\pi} \gamma(C,\tau)\, d\tau\, dC.$$

For $\varepsilon = 0$ the function F(C, τ) agrees with the distribution functions of stiff relatively coarse ellipsoidal particles in a shear flow of the suspension [8, 9].

Results of computations of the rheological characteristics of a dilute suspension of relatively coarse deformable particles on the dimensionless shear velocity $\varepsilon = K\mu/G$ performed on the basis of the theory elucidated above are given in Figs. 1-4.

Dependences of the characteristic viscosity of the suspension

$$[\mu] = \lim_{V\to 0} (\mu_a - \mu)/\mu V \tag{3.6}$$

on ε are shown in Figs. 1 and 2 for different values of the microstructure parameters. The μ_a is the effective viscosity of the suspension in (3.6). Thus, curves 1-4 in Fig. 1 are computed for $\eta/\mu = 2$ and $q_0 = 0.5$, 2, 3, 5, the straight lines 1-4 correspond to the characteristic viscosity of a dilute suspension of relatively coarse particles for the same q_0 as for the corresponding curves. The curves 1-5 in Fig. 2 are computed for $q_0 = 2$ and $\eta/\mu = 1$, 1.5, 2, 6, 11.

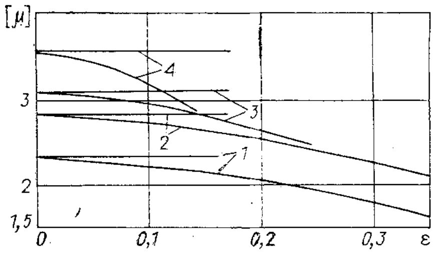

Fig. 1

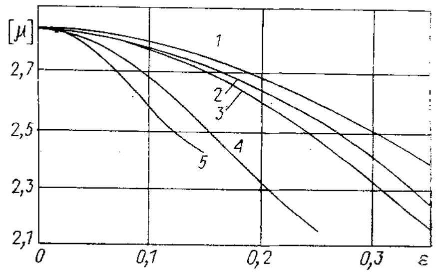

Fig. 2

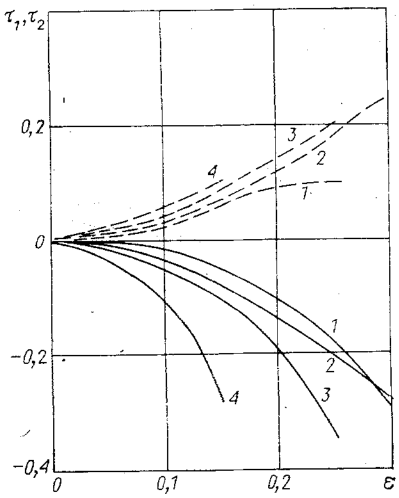

Fig. 3

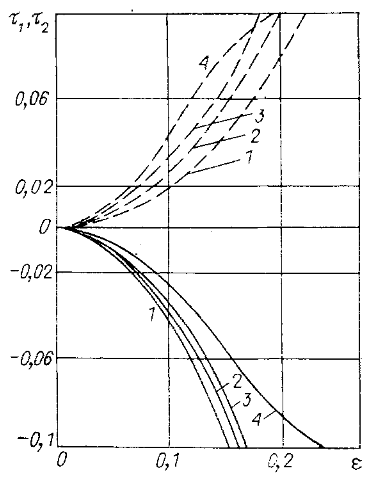

Fig. 4

Dependences of the dimensionless differences in the normal stresses $\tau_1 = (T_{xx} - T_{zz})/GV$, $\tau_2 = (T_{yy} - T_{zz})/GV$ on ε are presented in Figs. 3 and 4 (τ_1 are solid lines and τ_2 are dashed). The curves 1-4 in Fig. 3 correspond to $\eta/\mu = 2$ and $q_0 = 0.5, 2, 3, 5$. Curves 1-4 in Fig. 4 are obtained for $q_0 = 2$ and $\eta/\mu = 1, 1.5, 2, 6$.

The results of computations presented in the figures show that a dilute suspension of relatively coarse deformable particles (in contrast to a dilute suspension with relatively coarse stiff particles) in a simple shear flow has a non-Newtonian behavior, the presence of the difference in the normal stresses, the dependence of the rheological characteristics (the effective viscosity of the suspension and the difference in the normal stresses) on the shear velocity. The appearance of anomalous properties of the dilute suspension depend substantially on the internal viscosity and elasticity of the particle material and the viscosity of the dispersion medium.

Upon realization of the passage to the limit to the case of a suspension of solid spheres in the theory represented, the characteristic viscosity of the suspension is 2.5, which agree with the known Einstein formula for the viscosity of a dilute suspension of stiff spheres.

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CONVECTIVE INSTABILITY IN A MEDIUM WITH SPIRAL TURBULENCE

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In the papers of Moiseev, Sagdeev, Tur, et al. (see [1] and the literature cited there), the generation of large-scale convective structures on a background of spiral turbulence was considered and the relevant equations were obtained and analyzed. It was assumed that the turbulence is homogeneous and isotropic but does not possess reflection invariance. In this model random perturbations are amplified, which can lead to generation of large-scale vortices. This situation was studied in [1] for the example of a plane-parallel layer of incompressible liquid heated from below. Simplified boundary conditions were assumed in order to obtain an analytic solution. It was shown that as the spirality increases the minimum critical Rayleigh number decreases and the horizontal dimensions of the convection cells increase. At a critical value of the spirality, the structure of the convective flow change completely and a vortex is formed whose dimensions are determined by the external conditions of the problem such as inhomogeneities in the horizontal direction.

In the present paper the equations of [1] are used to analyze the convective instability in an infinite horizontal layer and in a disk heated from below in the linear theory.

The equations describing convection for large-scale disturbances in the presence of spiral turbulence have the form [1]

$$\frac{\partial \mathbf{u}}{\partial t} = -\frac{\nabla p}{\rho_0} + \nu \Delta u + \beta g \theta \mathbf{e} + \beta g A \lambda \mathbf{f}, \quad \frac{\partial \theta}{\partial t} = A(\mathbf{eu}) + \chi \Delta \theta, \quad \operatorname{div} \mathbf{u} = 0,$$
$$\mathbf{f} = \mathbf{e}(\mathbf{e} \operatorname{rot} \mathbf{u}) - (\mathbf{e}\nabla)[\mathbf{eu}], \ \mathbf{e} = (0, 0, 1).$$

Here ν and χ are the turbulent viscosity and thermal conductivity. Because these quantities are nearly equal to one another [1-3], we assume $\nu = \chi$. The coefficient λ is associated with the spirality of the turbulence. The rest of the notation is standard [2].

We transform to dimensionless variables using as scales of measurement [2]: $x_0 = H$ (height of the liquid layer) for length, $t_0 = H^2/\nu$ for time, $u_0 = \nu/H$ for velocity, $p_0 = \rho_0\nu^2/H^2$ for pressure, and $T_0 = AH$ for temperature. Then

$$\partial \mathbf{u}/\partial t = -\nabla p + \Delta \mathbf{u} + \mathrm{Ra}\, \theta \mathbf{e} + \mathrm{Ra}\, S \mathbf{f}, \ \partial\theta/\partial t = (\mathbf{eu}) + \Delta\theta,$$
$$\operatorname{div} \mathbf{u} = 0, \ \mathbf{f} = \mathbf{e}(\mathbf{e} \operatorname{rot} \mathbf{u}) - (\mathbf{e}\nabla)[\mathbf{eu}], \ \mathbf{e} = (0, 0, 1), \tag{1}$$

where Ra is the Rayleigh number and S is a coefficient connected with the spirality of the turbulence ($\mathrm{Ra} = \beta g A H^4/\nu^2$, $S = \lambda\nu/H^3$).

We consider an infinite horizontal liquid layer included between two planes $z = 0$ and $z = 1$. Then $\mathbf{u} = (u, v, w)$ and θ are given by

$$u = u'(z) \sin kx \exp(\gamma t), \ v = v'(z) \sin kx \exp(\gamma t),$$
$$w = w'(z) \cos kx \exp(\gamma t), \ \theta = \theta'(z) \cos kx \exp(\gamma t).$$

In the case of a cylindrical layer (disk) $\cos kx$ is replaced by the Bessel function of order zero $J_0(kr)$ and $\sin kx$ is replaced by the Bessel function of order one $J_1(kr)$. The results given below do not change in this case. Putting these substitutions into (1) we find

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